METHOD OF DETERMINING DISPLACEMENT OF THE CARDIAC ELECTRICAL DIPOLES BASED ON EXTINCTION OF POTENTIALS

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Two systems of leads, each with four electrodes, are proposed which together with a "corrected" orthogonal system of vector cardiographic leads, can be used to obtain equations for calculating spatial displacement of the equivalent electrical dipole of the heart. Results of a simple theoretical calculation and of an experiment on an electrophysical model illustrating the method are given.

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Modern corrected orthogonal systems of leads designed for accurate measurement of three components of the resultant dipole moment of the electric generator of the heart are insensitive to changes in position of elementary sources of current in space or, in the terms of a multipolar equivalent generator, to components of all multipoles of the heart above the first order [2].

However, convincing evidence has been obtained to show that three independent signals (in the case of corrected orthogonal systems, three components of the dipole moment) are insufficient to describe all accessible information concerning the electrical state of the heart [6, 9-11]. To make good the lost information which may be useful for the topical diagnosis of heart diseases, as well as dipole components, it is desirable to measure certain components of multipoles of higher orders.

In this paper we accept the hypothesis concerning the structure of the cardiac generator according to which all sources of current in the myocardium can be reduced to a single moving dipole [1]. This assumption appears justified because potentials on the surface of the body from a wave of excitation, in the form of an electrical double layer with an approximately constant dipole moment per unit area of surface, moving about in the myocardium, are approximated with sufficient accuracy by potentials from a single dipole generator, changing its intensity, orientation, and point of application [5, 7]. This assumption is also convenient because it provides for the addition of a few new measurements to the three measured components of the cardiac dipole: three coordinates of position of the dipole in space or two coordinates of its position in one plane.

To measure the higher components of an equivalent cardiac generator, the system of two leads (I and II) shown in Fig. 1 is proposed. Lead I contains four electrodes located at points 1 $(-\rho_1, 0, 0)$, 2 $(\rho_2, 0, 0)$, 3 $(0, 0, -\rho_3)$, and 4 $(0, 0, \rho_4)$ along the x and z axes of a rectangular system of coordinates x, y, z, with origin at the hypothetical geometric center of the heart; lead II contains four electrodes located similarly to those of lead I at points 5 $(-\rho_5, 0, 0)$, 6 $(\rho_6, 0, 0)$, 7 $(0, 0, -\rho_7)$, and 8 $(0, 0, \rho_8)$ of a rectangular system of coordinates x', y, z', turned through an angle $\pi/4$ around the y axis relative to the system of coordinates x, y, z. All measuring electrodes of the leads thus lie in a transverse plane passing through the center of the heart. Electrodes 1 and 2, 3 and 4, 5 and 6, 7 and 8 are connected by potentiometers whose resistance is much greater than the resistance of the body between any two electrodes of the leads. Signals of the leads taken from moving contacts of the potentiometers are given by:

$$\begin{aligned} \varphi_{I} &= \varphi_3(1-n_z) + \varphi_4 n_z - \varphi_1(1-n_x) - \varphi_2 n_x, \\ \varphi_{II} &= \varphi_7(1-n_{z'}) + \varphi_8 n_{z'} - \varphi_5(1-n_{x'}) - \varphi_6 n_{x'}, \end{aligned} \qquad (1)$$

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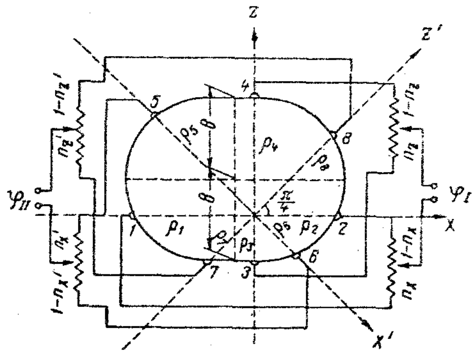

Fig. 1. Diagram of leads. Electrodes located in a transverse plane passing through the geometric center of the heart. Explanation in text.

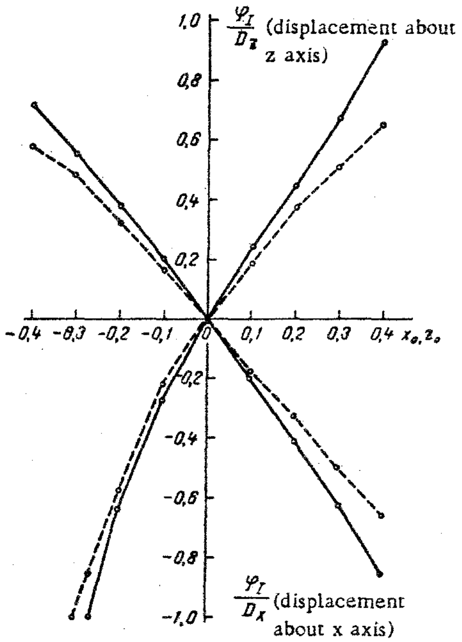

Fig. 2. Signal of lead I as a function of displacement of dipole about x and z axes. Continuous line gives results of calculation; broken line results of experiment on model. Explanation in text.

where φ_1, φ_2, φ_3, φ_4, φ_5, φ_6, φ_7, and φ_8 represent potentials at corresponding points of measurement and n_x, n_z, n'_x, and n'_z represent the relative resistances of the potentiometer arms.

A signal from any lead can be expressed correct to two places of decimals as

$$\varphi(t) = L_i H^i(t) + L_{ij} H^{ij}(t), \quad i, j = x, y, z, \tag{2}$$

where $H^i(t)$ is the vector of the heart, $H^{ij}(t)$ the quadripole tensor of the heart, L_i the lead vector, and L_{ij} the lead tensor of the second rank [4]. If at any actual moment of time the cardiac generator is represented by one point dipole with components D_x, D_y, D_z, applied at the point (x_0, y_0, z_0) at a distance $r_0 = (x_0^2 + y_0^2 + z_0^2)^{1/2}$ from the origin of the coordinates (0, 0, 0), where it has been assumed that the equivalent multipolar generator of the heart is located, the cardiac tensors can be described by

$$H^i = \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix},$$

$$H^{ij} = \begin{vmatrix} \frac{2}{3}(2D_x x_0 - D_y y_0 - D_z z_0) & D_x y_0 + D_y x_0 & D_x z_0 + D_z x_0 \\ D_x y_0 + D_y x_0 & \frac{2}{3}(2D_y y_0 - D_x x_0 - D_z z_0) & D_y z_0 + D_z y_0 \\ D_x z_0 + D_z x_0 & D_y z_0 + D_z y_0 & \frac{2}{3}(2D_z z_0 - D_x x_0 - D_y y_0) \end{vmatrix}. \tag{3}$$

The corresponding lead tensors will be of the form:

$$L_i = \begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix}, \quad L_{ij} = \begin{bmatrix} l_{xx} & l_{xy} & l_{xz} \\ l_{yx} & l_{yy} & l_{yz} \\ l_{zx} & l_{zy} & l_{zz} \end{bmatrix}, \tag{4}$$

where $l_{xy} = l_{yx}$, $l_{xz} = l_{zx}$, $l_{yz} = l_{zy}$, and $l_{xx} + l_{yy} + l_{zz} = 0$. For convenience in the calculations the components of the dipole moment are expressed as a ratios to a constant value $4\pi\gamma_0 b^2$, where γ_0 represents the mean specific conductivity of the body and b the characteristic length to which all coordinates and geometric measurements are related. The values of n_x, n_z, n'_x, and n'_z are chosen from the conditions of complete compensation or "extinction" of the potential from the dipole part of the equivalent generator at the moving contacts of the corresponding potentiometers; accordingly

$$n_x = \frac{l_{x1}}{l_{x1} - l_{x2}}, \quad n_z = \frac{l_{z3}}{l_{z3} - l_{z4}},$$
$$n_{x'} = \frac{l_{x'5}}{l_{x'5} - l_{x'6}}, \quad n_{z'} = \frac{l_{z'7}}{l_{z'7} - l_{z'8}}. \tag{5}$$

Calculations of the electrical field for simple theoretical models and experiments on electrophysical models of the human body showed that when the above conditions of geometric placing of the electrodes on the body surface and adjustment of the potentiometers are satisfied, all the lead parameters are negligibly small compared with components of the second rank tensor l_{xxI} and l_{zzI} for lead I and l_{zxII} and l_{xzII} for lead II, that is:

$$L_{iI} \approx \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad L_{ijI} \approx \begin{bmatrix} l_{xxI} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & l_{zzI} \end{bmatrix}, \quad L_{iII} \approx \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad L_{ijII} \approx \begin{bmatrix} 0 & 0 & l_{xzII} \\ 0 & 0 & 0 \\ l_{zxII} & 0 & 0 \end{bmatrix} \tag{6}$$

Substitution of expressions (3) and (6) in the general expression for the lead signal (2) gives the following approximate expressions for signals of leads I and II:

$$\begin{aligned} \varphi_I &= 2l_{xxI} D_x x_0 + 2l_{zzI} D_z z_0, \\ \varphi_{II} &= 2l_{zxII} D_z x_0 + 2l_{xzII} D_x z_0. \end{aligned} \tag{7}$$

these expressions can be regarded as a system of linear algebraic equations relative to unknown coordinates of the position of the dipole generator in the transverse plane x_0 and z_0 for each moment of time. To solve the system it is necessary to know the lead parameters l_{xxI}, l_{zzI}, l_{xzII}, and l_{zxII}, and the components of the dipole moment D_x and D_z (which can be measured by a well corrected orthogonal system of leads).

For a theoretical assessment of the values of l_{xxI}, l_{zzI}, l_{zxII}, and l_{zxII}, the following simple model is taken: the conductor in which the cardiac generator functions is represented by a homogeneous medium of infinite extent with a specific conductivity γ_0. Parameters of a unipolar lead from any point (x, y, z) lying outside the region of the heart at a distance of $\rho = (x^2 + y^2 + z^2)^{1/2}$ from the origin of the coordinate can be described for such a model as [12]:

$$L_i = \frac{1}{\rho^3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad L_{ij} = \frac{1}{2\rho^5} \begin{bmatrix} 2x^2 - y^2 - z^2 & 3xy & 3xz \\ 3xy & 2y^2 - x^2 - z^2 & 3yz \\ 3xz & 3yz & 2z^2 - x^2 - y^2 \end{bmatrix}. \tag{8}$$

For the body of a particular person ($\rho_1 = 1.47$, $\rho_2 = 1.07$, $\rho_3 = 0.70$, $\rho_4 = 1.30$, $\rho_5 = 1.59$, $\rho_6 = 0.82$, $\rho_7 = 0.94$, and $\rho_8 = 1.42$), Eqs. (8), (1), and (5) give values for the relative arms of the potentiometers $n_x = 0.35$, $n_z = 0.78$, $n'_x = 0.21$, and $n'_z = 0.70$, with the following concrete form of the system of equations (7):

$$-1.98 D_x x_0 + 2.51 D_z z_0 = \varphi_I, \quad 1.70 D_z x_0 + 1.70 D_x z_0 = \varphi_{II}. \tag{9}$$

Experimental tests of the lead parameters were carried out on an electrophysical model reproducing the body of the same person and allowing for electrical nonhomogeneity of the body caused by the low specific conductivity of the lungs compared with the other tissues [3]. As a result of these experiments parameters for adjustment of the potentiometers $n_x = 0.40$, $n_z = 0.76$, $n'_x = 0.27$, and $n'_z = 0.63$ and the following system of equations were obtained:

$$-1.65 D_x x_0 + 2.03 D_z z_0 = \varphi_I, \quad 1.74 D_z x_0 + 1.74 D_x z_0 = \varphi_{II}. \tag{10}$$

Changes in the signal in lead I during movement of the dipole, lying in the direction of the x and z axes, about these axes, are shown in Fig. 2 as an example. The shape of the curves shows some disturbance of linearity of the relationship, but this nonlinearity is evidently systematic in character and leads only to a change in the scale of measurement for certain directions of displacement of the dipole. In conclusion, it may be noted that the accuracy of determination of coordinates of the cardiac electrical dipole in the transverse plane by the method described well depend on stability of the parameters of leads I and II when used with different subjects, and also on the accuracy of measurement of components of the dipole moment. Further investigations are required to estimate the accuracy of the method.

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